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LETTER TO THE EDITOR

Critical behaviour of a model with NNN interaction

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Abstract. A one-dimensional quantum model with next-nearest-neighbour interaction is studied. Under the long-wavelength approximation, we predict that this model belongs to the same universality class as the Ising model. This conclusion is checked with a numerical solution which is consistent with the prediction in both critical point and critical exponent.

Recently, we studied a quantum model with NNN interaction whose Hamiltonian takes on the form of (Xu and Tang 1989)

$$H = g\sum_n \sigma_n^z - \sum_n \sigma_n^x\sigma_{n+1}^x - \frac{\lambda}{2}\sum_n (\sigma_n^x\sigma_{n+2}^x + \sigma_n^y\sigma_{n+2}^y). \tag{1}$$

In our previous work, we predicted that this model belongs to the same universality class as the Ising model and gave the critical point condition

$$g - 1 - \lambda = 0. \tag{2}$$

We should notice that our result depends on the long-wavelength approximation, which is feasible only under the condition that this model does have critical phenomenon. So, we should ascertain the existence of critical point to guarantee our conclusion.

As is well known, a one-dimensional quantum chain is equivalent to a two-dimensional infinitely long classical strip with finite width. It has been proved that for two-dimensional classical systems on infinitely long strips of finite width N, the scaling dimension $\chi = \eta/2$ relates to the energy gap between the ground state and the first excited state (Cardy 1984, Alcaraz and Barber 1987, von Gehlen *et al* 1985) as

$$\Delta E = E_1 - E_0 = \pi\eta\xi/N \tag{3}$$

here ξ is a coefficient which depends on g and λ. We can use this property to determine whether a model has critical phenomenon. That is, if a model has critical phenomenon, there must be a critical point at which η will approach a fixed value when $N\to\infty$, and the critical point can be found. In fact, all we should do is calculate the energy gap ΔE with different N (for a corresponding quantum chain, N is the number of lattice sites).

Many numerical methods have been developed to calculate the energy gap between the ground state and the first excited state of a one-dimensional quantum chain (Hamer and Barber 1981, Duxbury and Barber 1982). But we take Lanczos' approach which seems to be the most efficient one (Roomany *et al* 1980).

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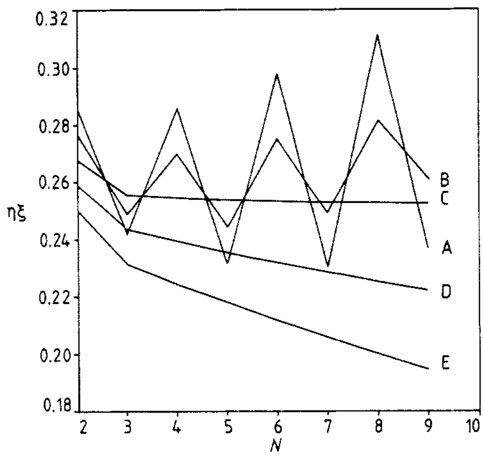

Figure 1. Asymptotic behaviour of $\eta\xi$ with $\lambda = 0.01$ and A $g = 1.05$, B $g = 1.03$, C $g = 1.01$, D $g = 0.99$, E $g = 0.97$. Lines are drawn to illustrate the asymptotic behaviour but only nine points exist on each line in reality.

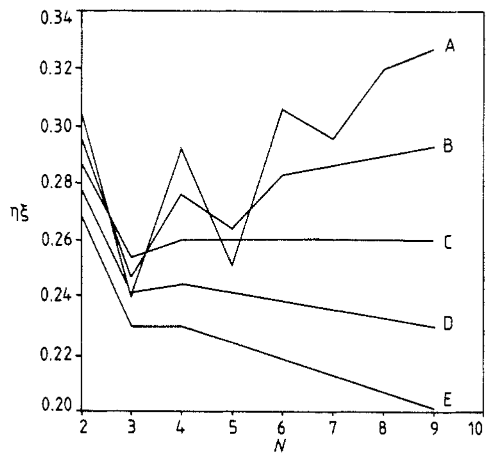

Figure 2. Asymptotic behaviour of $\eta\xi$ with $\lambda = 0.05$ and A $g = 1.09$, B $g = 1.07$, C $g = 1.05$, D $g = 1.03$, E $g = 1.01$.

Here we give our numerical results in figures 1-3 and tables 1-8. The largest system used in our calculations is a chain of nine sites. The exponents of both of Ising model ($\lambda = 0$) and the model of (1) with different λ have accuracies of a percent at these sites. While the Ising model does have critical phenomenon, so these results convince us of the existence of critical phenomenon in the model of (1). Furthermore, the critical points are consistent with the critical condition of (2). In addition to the relation of (3), conformal theory predicts that the ground-state energy per site should approach its bulk limit e_0, namely (Nightingale and Blöte 1983, Blöte *et al* 1986)

$$E_0/N = e_0 - \pi c\xi/6N^2 + \mathrm{O}(1/N^2) \tag{4}$$

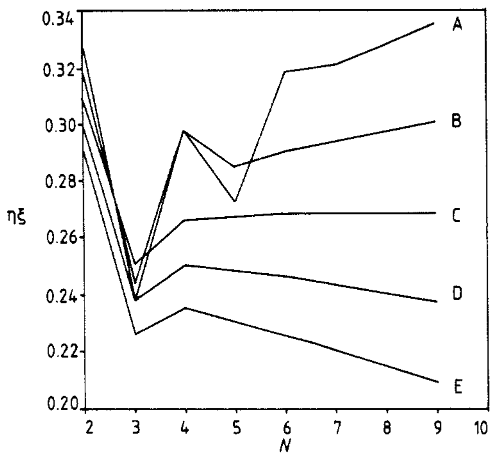

Figure 3. Asymptotic behaviour of $\eta\xi$ with $\lambda = 0.10$ and A $g = 1.14$, B $g = 1.12$, C $g = 1.10$, D $g = 1.08$, E $g = 1.06$.

where c is the central charge of the conformal class to which this model belongs. We can calculate both e_0 and $c\xi$, and hence the value of c/η, which identifies the universality class of the model. For example, c/η is 2 for the Ising class ($c = 0.5$, $\eta = 0.25$, table 1 and table 2). From table 3 to table 8, we can see that for the model of (1) with $\lambda \neq 0$, c/η also approaches 2, so we can reasonably say that this model really belongs to Ising universality class. This result is consistent with our previous conclusion with respect to both critical point and critical exponent.

Table 1. Mass gap amplitude as a function of lattice size; $\lambda = 0.00$, $g = 1.00$ (Ising model).

N	E_0	E_1	$N(E_1 - E_0)$	$\eta\xi$
2	−2.973 21	−2.000 00	1.946 42	0.619 56
3	−4.251 83	−3.726 93	1.574 70	0.501 24
4	−5.590 70	−5.173 21	1.669 96	0.531 56
5	−6.925 38	−6.590 53	1.676 50	0.533 65
6	−8.270 79	−7.990 31	1.682 88	0.535 68
7	−9.620 31	−9.379 75	1.683 92	0.536 01
8	−10.973 10	−10.762 61	1.683 92	0.530 61
9	−12.328 08	−12.141 04	1.683 36	0.535 83

Table 2. Value of c/η as a function of lattice size; $\lambda = 0.00$, $g = 1.00$ (Ising model) ($b = \pi c\xi/6$).

$N1$	$N2$	e_0	b	$c\xi$	c/η
2	3	1.361 81	0.499 16	0.953 33	1.901 93
3	4	1.372 47	0.403 23	0.770 11	1.448 78
4	5	1.362 93	0.555 96	1.061 81	1.989 70
5	6	1.363 24	0.548 26	1.047 10	1.954 71
6	7	1.362 88	0.561 12	1.071 66	1.999 33
7	8	1.362 84	0.562 91	1.075 08	2.053 25
8	9	1.362 82	0.564 40	1.077 92	2.011 68

Table 3. Mass gap amplitude as a function of lattice size; $\lambda = 0.01$, $g = 1.01$.

N	E_0	E_1	$N(E_1 - E_0)$	$\eta\xi$
2	−2.842 60	−2.000 00	1.685 20	0.536 42
3	−4.025 02	−3.490 11	1.604 73	0.510 80
4	−5.262 42	−4.862 60	1.599 28	0.509 70
5	−6.517 29	−6.198 55	1.593 70	0.507 29
6	−7.781 41	−7.516 35	1.590 36	0.506 32
7	−9.050 75	−8.823 87	1.588 16	0.505 53
8	−10.323 34	−10.124 99	1.586 80	0.505 09
9	−11.598 07	−11.421 88	1.585 71	0.504 75

Table 4. Value of c/η as a function of lattice size; $\lambda = 0.01$, $g = 1.01$.

$N1$	$N2$	e_0	b	$c\xi$	c/η
2	3	1.277 97	0.573 31	1.094 94	2.041 20
3	4	1.282 09	0.536 26	1.024 18	2.011 87
4	5	1.281 86	0.539 87	1.031 08	2.032 52
5	6	1.282 00	0.536 43	1.024 51	2.023 80
6	7	1.282 06	0.534 31	1.020 46	2.018 59
7	8	1.282 10	0.532 45	1.016 90	2.013 31
8	9	1.282 11	0.531 53	1.015 15	2.011 19

Table 5. Mass amplitude as a function of lattice size; $\lambda = 0.05$, $g = 1.05$.

N	E_0	E_1	$N(E_1 - E_0)$	$\eta\xi$
2	−2.900 00	−2.000 00	1.800 00	0.572 96
3	−4.125 46	−3.594 79	1.592 01	0.506 75
4	−5.407 72	−5.000 00	1.630 88	0.519 13
5	−6.698 39	−6.372 04	1.631 75	0.519 40
6	−7.998 19	−7.726 19	1.632 00	0.519 48
7	−9.303 03	−9.070 00	1.631 20	0.519 23
8	−10.611 12	−10.407 34	1.630 24	0.518 92
9	−11.921 37	−11.740 35	1.629 18	0.518 58

Table 6. Value of c/η as a function of lattice size; $\lambda = 0.05$, $g = 1.05$.

$N1$	$N2$	e_0	b	c/η
2	3	1.3153	0.5389	1.796
3	4	1.3221	0.4777	1.800
4	5	1.3179	0.5445	2.002
5	6	1.3179	0.5437	1.999
6	7	1.3178	0.5465	2.010
7	8	1.3179	0.5466	2.012
8	9	1.3178	0.5468	2.014

Table 7. Mass amplitude as a function of lattice size; $\lambda = 0.10$, $g = 1.10$.

N	E_0	E_1	$N(E_1 - E_0)$	$\eta\xi$
2	−2.828 43	−2.000 00	1.656 86	0.527 39
3	−4.000 00	−3.464 10	1.607 70	0.511 75
4	−5.226 25	−4.828 43	1.591 28	0.506 52
5	−6.472 14	−6.155 37	1.583 85	0.504 16
6	−7.727 41	−7.464 10	1.579 86	0.502 89
7	−8.987 92	−8.762 58	1.577 38	0.502 10
8	−10.251 66	−10.054 68	1.575 84	0.501 61
9	−11.517 54	−11.342 56	1.574 82	0.501 28

Table 8. Value of c/η as a function of lattice size; $\lambda = 0.10$, $g = 1.10$.

$N1$	$N2$	e_0	b	$c\xi$	c/η
2	3	1.268 63	0.582 35	1.112 21	2.108 89
3	4	1.272 86	0.550 71	1.051 78	2.076 48
4	5	1.272 86	0.539 31	1.030 01	2.043 01
5	6	1.273 07	0.533 97	1.019 81	2.027 89
6	7	1.273 15	0.531 01	1.014 15	2.019 83
7	8	1.273 19	0.529 16	1.010 62	2.014 75
8	9	1.273 21	0.527 80	1.008 03	2.010 91

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